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A FEW QUESTIONABLE POINTS IN THE HISTORY OF MATHEMATICS

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MOST of the professional mathematical historians have been basement builders and many of our general histories of mathematics remind one of the church buildings which consist of a basement roofed over while funds for completing the structure are being awaited. In some cases, such as Cantor's noted *Vorlesungen über Geschichte der Mathematik,* the basement is not even roofed over.

In fact, the work of Cantor might remind one in a mild way of the following statement in the Scriptures: "This man began to build and was not able to finish." If it is true that about *fifteen* volumes would be required in order to cover the developments of the nineteenth century as completely as Cantor covered the period up to the beginning of this century, as is suggested in the preface to Volume 1 of Dickson's "History of the Theory of Numbers," 1919, it results that Cantor did not complete one-fifth of the job of writing a general history of mathematics up to the end of his scientific activity.

It seems questionable whether a basement history, even when the basement is roofed by slight attention to the developments of the nineteenth and the twentieth centuries, is the most suitable history to place in the hands of the young student. Present day activities in mathematics have received entirely too little attention even on the part of the students who specialize in this subject.

A considerable number of questions in the history of mathematics have been answered differently by different writers and hence this subject offers unusual opportunities for the exercise of judgment and for argumentation. To some this may appear to be an attractive feature since disputation has long been recognized as a useful educational exercise and elementary mathematics presents comparatively few questions to which different answers have been given by good recent writers. Hence the student of this subject is inclined to confine his attention too closely to questions which can be answered definitely.

It is not the object of the present paper to give a definite answer to some of the questions which have been in dispute for a long time but rather to direct attention to a few more questions which seem to be open to dispute, in the hope that the interest in these questions may thereby be increased and the interest in the history of our subject may thus be fostered. In the main the present writer will present here arguments exhibiting a point of view which is not in accord with the one presented in the second edition of Cajori's "History of Mathematics," 1919, and our references to pages relate to this work.

On page 142 it is stated that "the foremost French mathematician before Vieta was Peter Ramus (1515-1572), who perished in the massacre of St. Bartholomew." It can not be assumed that the fact that Ramus perished during the massacre of St. Bartholomew constitutes a a claim to mathematical fame since thousands of others were then slain, but one consults the index of Cajori's history in vain for other reasons for calling Ramus the foremost French mathematician before Vieta.

It is interesting to note that in the history of many a country there is a record of some mathematician who is very much better known than any of his predecessors in the same country. As instances we may cite Newton in England, Leibniz in Germany, Napier in Scotland, Abel in Norway, etc. In some cases very little is known about any of the predecessors of such a man in the country in question. For instance, G. A. Gibson stated that before Napier (1550-1617) Scotland made not a single contribution to mathematical science [1]. In case more is known about the predecessors of such a man it is a question of some interest to inquire into the relative merits of their contributions.

Hence one is naturally interested in knowing something about the work of the French predecessors of Vieta who is doubtless much better known than any of these predecessors. Among these there are, in addition to Ramus, such favorably known men as N. Oresme and N. Chuquet. The reader who recalls the many references to the works of the last two mathematicians (e. g., on page 14 of tome 3, volume 3, of the "Encyclopédie des Sciences Mathématiques" G. Eneström notes that a work by Oresme serves as a graphic preamble to the introduction of analytic geometry) will naturally wonder why Ramus is placed ahead of them in both editions of Cajori's "History of Mathematics."

It is true that Ramus is better known in general than Oresme or Chuquet, but Ramus is known principally on account of his attacks on the accepted views of his day and not on account of his contributions towards the advancement of mathematics. In mathematics he also exhibited his quarrelsome disposition but he failed even to understand the more subtle points involved in some of the mathematical methods which he attacked. He emphasized the importance of teaching the practical methods of calculation employed by the merchants of the street and his mathematics was of the business college type rather than of the university type.

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[1] "Napier Tercentenary Celebration Handbook," 1914, p. 2.

His contention that the method of giving a collection of definitions first, as is done in Euclid's "Elements," is unnatural since a forest was not created by growing the roots of the trees first may have had considerable influence on later textbooks on elementary mathematics. It is, however, a question whether a quarreling and quarrelsome dialectician, such as Ramus was, should be placed ahead of Oresme and Chuquet as a mathematician even if his activities had a wholesome influence on mathematical instruction and may have been largely responsible for the early and radical departure from Euclid's "Elements" on the part of French textbooks on geometry.

As the names of Oresme and Chuquet are prominent in the history of exponents in elementary mathematics, the former having used fractional exponents and the latter the exponent zero, their work naturally calls in question the following statement found on page 149: "It is one of the greatest curiosities in the history of science that Napier constructed logarithms before exponents were used." The notion of exponents and not the formal use of them in the modern way is related to the development of logarithms, and this notion was much older even than the work of Oresme, who lived more than two centuries before Napier.

In view of the great mathematical influence of the Ecole Normale of Paris it may be of interest to refer to the statement found in various places to the effect that its first students were young pupils. For instance, on page 256, it is stated that "at the establishment of the *Ecole Normale* in 1795 in Paris, he (Lagrange) was induced to accept a professorship. Scarcely had he time to elucidate the foundations of arithmetic and algebra to young pupils, when the school was closed."

While the term "young pupils" is not very definite, yet few people would be likely to associate it with students whose ages varied from 21 to 66. In fact, according to "Le Centenaire de l'Ecole Normale," 1895, page 125, nearly half of these "young pupils" were from 30 to 60 years old, and among them was Bougainville, a celebrated navigator, who was 66 years old. The law prescribed that none of these students should be less than 21 years old but it did not fix an upper limit to their ages as the best prepared available students were desired.

In view of the great influence which this ephemeral experiment had on the teaching of mathematics and other sciences in the secondary schools of France and the fact that the professors of mathematics (Lagrange, Laplace and Monge) were eminent mathematicians, it is of interest to know that these early students, who were paid to come to Paris from various parts of France, were not "young pupils" in the sense in which this term is commonly understood, but were, in the main, mature men who could derive much profit from a profound presentation of the elements of various educational subjects. Some of

Lagrange's lectures prepared for these students were translated by T. J. McCormack and published in 1901 under the title "Lectures on Elementary Mathematics by Joseph Louis Lagrange," The Open Court Publishing Company.

It may be noted here that the official journal of this normal school during the first brief period of its existence was entitled *Séances des Ecoles Normales*, and not *Journal des Ecoles Normales* as is stated in various places including the *Encyklopädie der Mathematischen Wissenschaften*, volume 3, page 519, and on page 274 of the history noted above. In the latter work we find also on page 204 the title *Transactions of the London Mathematical Society* instead of *Proceedings* of this society. In this case the title is the more misleading since the number of the volume is also incorrectly stated as 20 instead of 22.

It is true that the said *Séances* really were a journal and the said *Proceedings* really involve what is commonly called transactions, but the work of the beginner is apt to be greatly increased by a failure to give exact references and it is the beginner who should be especially encouraged to look up references. If such a reader fails to find a journal which bears the exact title given in the reference he seldom looks further.

While a study of the history of mathematics doubtless tends towards the formation of clearer mathematical concepts it is evidently necessary for the student of this history to distinguish carefully between the good and the bad in ancient methods. Some of the ancient methods which may appear to be praiseworthy for the time when they originated would be questionable and perhaps even intolerable if they were used in our modern textbooks. Possibly the ancient Greek proof of the fact that $\sqrt{2}$ is not a rational number belongs to this category since it is special and does not appear any easier than the following more general proof, which is based upon the elementary fact that if a rational fraction is reduced to its lowest terms than every integral power of this fraction is also in its lowest terms.

Suppose that $\sqrt[n]{m}=c/d$, where c/d is reduced to its lowest terms and d is not equal to 1. By raising both members of this equation to the n^{th} power it results that $c^n/d^n=m$, and since c^n/d^n is reduced to its lowest terms m can not be an integer. This known proof establishes at one stroke the existence of an infinite number of irrational numbers if we assume the existence of at least one real n^{th} root of every positive integer. To the extent that a knowledge of the history of mathematics leads us to prefer old historic methods to equally simple more general methods it is positively injurious.

Young teachers who study the history of mathematics with the laudable purpose of increasing their efficiency in the class-room should bear in mind that there are exceptions to the rule that the mathematical development of the student is similar to the mathematical development of the human race. The modern student can not afford to acquaint himself with all the special and crude methods of the ancients before becoming familiar with the more powerful modern methods. The history of our subject is useful to the teacher provided he uses it to suggest methods rather than to supply these methods.

One of the most important questionable points in a general history of mathematics is the emphasis, or the lack of emphasis, on mathematical insight into the questions under consideration. It is evident that statements which have no mathematical sense such as the following: "In 1869 C. F. Geiser showed that 'the projection of a cubic surface from a point upon it on a plane of projection parallel to the tangent plane at that point, is a quartic curve; and that every quartic curve can be generated in this way," which is found on page 318 of the history noted above, should be avoided as far as possible. Similarly, authors should aim to avoid statements which are apt to be misunderstood because additional data must be supplied before they have any real significance, such as the following: "Newton uses his formulas for fixing an upper limit of real roots; the sum of any even power of all the roots must exceed the same even power of any one of the roots," which is found on page 202 of the same work.

There are, however, many statements which are perfectly accurate and yet fail to bring out the real mathematical situation. As regards modern developments some such statements can scarcely be avoided in view of the fact that details would involve an almost endless amount of explanations, but such details can be more easily supplied as regards ancient mathematics. For instance, the following theorem relating to the addition of the digits of a positive integer is found in various general histories of our subject. If any three consecutive positive integers, of which the largest is divisible by 3, are added together there results a number which is either 6 or reduces to 6 by the successive addition of its digits. The full significance of this theorem becomes clear only after considering it as a special case of the theorem that numbers which are congruent modulo 9 constitute an invariant with regard to the operation of adding digits, and observing the connection between this theorem and the ancient rules relating to "casting out the 9's."

We shall direct attention here to only one more questionable historical statement which appears on page 175 of the history to which we referred several times above, and reads as follows: "The new feature introduced by Descartes was the use of *an equation with more than one unknown*, so that (in case of two unknowns) for any value of one unknown (abscissa), the length of the other (ordinate) could be computed." From the words in italics one would naturally infer that the emphasis was to be placed on the fact that Descartes used equations involving more than one unknown. On the contrary, the emphasis should be placed on the functional relation between the unknowns.

Equations with more than one unknown are very old in mathematics. In fact, it is well known that statements equivalent to such equations appear on one of the oldest fragments of papyri. In modern notation these equations have been expressed as follows:

$$x^2+y^2=100 \qquad y=\tfrac{3}{4}x.$$

A considerable part of the well known "Arithmetica" by Diophantus relates to equations in two unknowns and the Hindus used equations with more than one unknown, distinguishing them by colors, as the black, blue, yellow, red, or green unknown. As regards the expression of functional relations Descartes' work is well known to have been epoch-making.

In the present state of our knowledge of the history of mathematics it seems almost impossible to avoid questionable statements in works which aim to cover the entire field. The suggestions here offered relating to the other side of questions involved in various such statements may serve to arouse interest in a few important historical matters, especially on the part of those who enjoy the clash of views in a friendly combat.